

ANALYSIS OF SOFTENING RC ELEMENTS USING NON-LOCAL DIFFERENTIAL FIBER ELEMENT MODEL

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ABSTRACT

This paper presents a method of analysis for evaluating the softening behavior of RC members. In order to obtain mesh objective results, a non-local fiber element model is introduced. The resisting force of each section, in the non-local fiber element model, is calculated using a proposed differential hysteretic model, which is especially developed for modeling softening behavior under cyclic loading. The section hysteretic response is determined using microplane constitutive model for concrete and bi-linear elasto-plastic model for reinforcements. The calculated hysteretic behavior of sections is employed to scale the parameters of the differential hysteretic model. The proposed method is verified by the available experimental results. Close agreements between the numerical and experimental results are obtained. The method provides insight to the procedure of flexural localization. Also, due to computational efficiency of the method, it is possible to extend its use for analyzing RC frames with softening members.

Keywords: differential hysteretic model, fiber element model, non-local model, reinforced concrete flexural members, softening

1. INTRODUCTION

Fiber element models, used in this study, are among most promising nonlinear analysis models of frame structures. In these models load-displacement history of a number of sections along member length are traced in order to capture the gradual spread of nonlinearity. Fiber element models can be classified in two main groups: stiffness based and flexibility based models. As discussed by Zeris and Main [13] the conventional stiffness based models are unable to satisfy equilibrium along member length when softening occurs. They proposed a procedure to handle the situation. Later Neuenhofer and Filippou [7] modifying the model introduced by Spacone et al. [11], developed a general flexibility based model with the ability to strictly satisfy equilibrium.

It is well known that standard local finite element methods are inappropriate for analysis of softening structures. Softening engenders problems of lack of mesh objectivity and localization of the displacements into narrow bands whose width depends on the element size and tends to zero as the mesh refined. A simple and computationally efficient way to avoid aforementioned

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problems is to use non-local formulations. Bazant [1] and Jirasek and Bazant [4] postulated a series of general non-local models, in which stress at each point is a function of not only strain at that point, but also the strains in its vicinity. Application of these models to the flexural softening problems, in cases of existence of moment gradient, shows that these models are not able to properly predict the element behavior. To overcome this deficiency a non-local model was developed which is especially suitable for the case of non-homogenous stress fields.

To improve computational efficiency in the fiber element model, for evaluation of the section resisting force, hysteretic differential model is used. Conventional differential models, based on Bouc-Wen differential model, are not able to properly model the softening behavior in the cyclic loading. A hysteretic differential model is developed, which is capable of modeling the section behavior during cyclic loadings that enter softening.

To scale parameters of hysteretic model, appropriate constitutive models for concrete and reinforcements should be used to obtain the section moment-curvature behavior. In this paper the micro-plane model of Bazant et al. [2] for concrete and bilinear elasto-plastic model for reinforcements are used.

In the following sections, the procedure used for analysis of RC members with softening behavior is presented. This procedure includes evaluation of section load-displacement behavior, the proposed differential hysteretic model and non-local fiber element model.

2. DERIVATION OF SECTION MOMENT-CURVATURE BEHAVIOR

To obtain the section moment-curvature behavior, the micro-plane model of Bazant et al. [2] as constitutive model for concrete and bi-linear elasto-plastic model for reinforcements is used. In the following some points associated with the use of micro-plane model are discussed.

In compression reloading, after tension unloading the micro-plane model shows incorrect behavior. In the micro-plane model concrete enters compression incorrectly before the tensile cracks are closed, even without any stiffness degradation. The following steps are taken to avoid this problem: (1) All value of the all micro-planes variables in the gauss points, in the last converged compression status are stored, (2) A macroscopic constitutive law is used for concrete in tension, and (3) In compression reloading, after closing of the tensile cracks, the last converged compression state is used as initial condition. The stress-strain relationship in tension is taken linear up to the cracking and then employing the following exponential function

$$\sigma = f_{cr} \exp(-\lambda(\varepsilon - \varepsilon_{cr})) \quad (1)$$

in which f and ε are stress and strain of concrete layer, f_{cr} and ε_{cr} are the stress and strain corresponding to the crack initiation and λ is a positive constant. It is assumed that tensile unloading and reloading follow secant slope after tensile cracking.

Euler hypothesis of plane sections remain plane after deformation is assumed to obtain the section moment-curvature relationship. Since at this stage only the behavior of one section is considered, there are parallel coupling between concrete layers. Subsequently, there is no need to use any special measure, such as non-local models at this stage. Displacement control on curvature and load control on axial force is used to develop the section load-displacement relationship. The initial stiffness method is used for iterative solution of the equilibrium equations.

Concrete at each layer of the section is modeled as a three-dimensional Gauss point. Lateral equilibrium at each layer is imposed separately, this makes it is possible to take into account the

different confining condition at each layer. For each layer the shape and spacing of transverse reinforcement is considered by changing the amount of effective transverse reinforcement. It is assumed that the strains in the two transverse directions are equal. Therefore lateral equilibrium equation for each concrete layer (Gauss point) will be

$$(F_{ct}')_{avg} + \rho \sigma_{st}' A_{st}' = 0 \quad (2)$$

in which $(F_{ct}')_{avg}$ is the average of the two lateral forces in concrete layer i , and ρ denotes the degree of efficiency of passive confinement, σ_{st}' is the stress in the transverse reinforcements of layer i and A_{st}' is the area of the transverse reinforcement per unit length of the flexural member. The degree of efficiency of the passive confinement is defined as the ratio of the confined concrete area to the gross area of each layer. The method of Sheikh and Uzumeri [10] is used for calculating the confined concrete area in each layer. The method takes into account the shape and spacing of the transverse reinforcements. For transverse reinforcements bi-linear hysteretic model is used.

3. DIFFERENTIAL HYSTERETIC MODEL

After evaluating the section moment-curvature, a differential hysteretic model is used to simulate the section load-displacement. Based on the differential hysteretic model of Bouc-Wen, dimensionless rate equation for section resisting moment will be

$$\dot{M} = \alpha \dot{\varphi} + (1 - \alpha) \dot{z} \quad (3)$$

in which α is the ratio of post-yield to pre-yield stiffness, φ is dimensionless curvature and z is the hysteretic curvature and dot denotes derivative with respect to time. Using the Foliente [3] differential model, the rate equation for z is

$$\dot{z} = h(z) [1 - (\beta \operatorname{sgn}(\varphi) \operatorname{sgn}(z) + \gamma) |z|^n] \dot{\varphi} / \eta \quad (4)$$

where β , γ and n are the parameters of the hysteretic model, η is stiffness degradation parameter, $h(z)$ is the pinching function and sgn is the signum function. Assuming that sum of β and γ is equal to one, the pinching function $h(z)$ in unloading is equal to one (i.e., there is no pinching) and in loading and reloading is given by

$$h(z) = 1 - \delta_1 \exp(-(z \operatorname{sgn}(\varphi) - q_1)^2 / \delta_2^2) \quad (5)$$

in which δ_1 and δ_2 evolve as function of hysteretic dissipating energy u_e as follows

$$\delta_1(u_e) = \delta_{10} (1 - \exp(-q_2 u_e)); \quad \delta_2(u_e) = (q_3 + q_4 u_e)(q_5 + \delta_1) \quad (6a,b)$$

in these equations q_1, \dots, q_5 and δ_{10} are constants which control the spread and severity of pinching. The rate equation for hysteretic dissipating energy is

$$u_e = (1 - \alpha)z\varphi \quad (7)$$

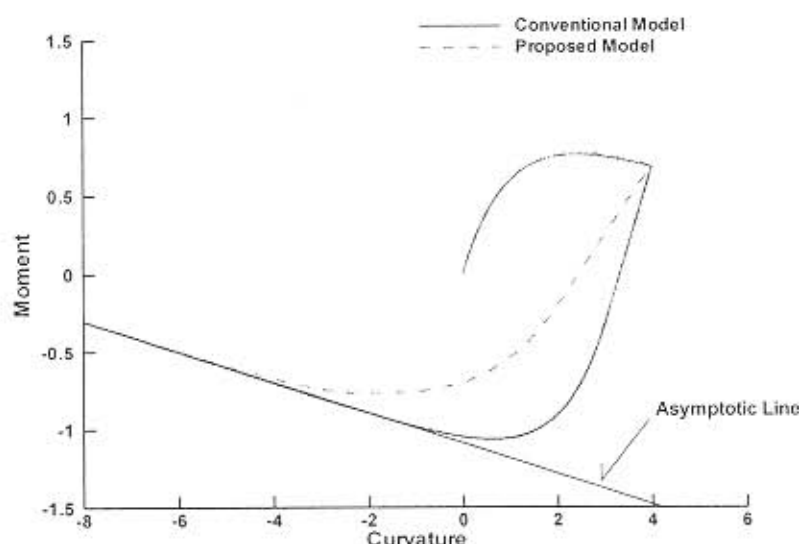


Figure 1. Behavior of the conventional and proposed differential models in cyclic loading

Elastic constitutive equation for axial force is used.

As can be seen in Figure 1 there are two asymptotic lines for $M-\varphi$ curve at large curvatures. In the conventional differential model when softening in the positive (negative) direction occurs, in load reversal there will be a fictitious increase in the strength in the negative (positive) directions. This behavior is not justified.

4. PROPOSED DIFFERENTIAL MODEL

To overcome this deficiency, the following evolution equation for η is introduced. From hereon it is assumed that n is equal to one. This restriction does not influence the capability of the model. The evolution equation for η will be

$$\eta = 21.2643\omega(\omega - .0504)(\omega^2 + .4568\omega + .1244)\exp(-a) \quad (8)$$

Note that when softening occurs, η remains constant. In unloading, a is

$$a = -1 + \frac{\log(1 + \text{sgn}(\varphi)(\gamma - \beta)z_0)}{\gamma - \beta} + \log\left(-\frac{\alpha}{1 - \alpha}\right) \quad (9)$$

while for loading or reloading

$$\alpha = -1 + \log\left(-\frac{\alpha}{(1-\alpha)(1-z_0)}\right) \quad (10)$$

Other variables in (8) are defined as follow

$$\omega = c \exp(\alpha); c = \text{sgn}(\varphi)(M_0 + (1-\alpha)(\text{sgn}(\varphi) - z_0) - M_p)/\alpha \quad (11a,b)$$

where M_0 and z_0 are the values of moment and hysteretic curvature in the beginning of load-step and M_p is the peak resisting moment of the section determined as follow.

$$M_p = M_p^+; (d\varphi > 0) \quad (12a)$$

$$M_p = M_p^-; (d\varphi < 0) \quad (12b)$$

Before occurrence of softening

$$M_p^+ = -\log(-\alpha/(1-\alpha)) + 1; M_p^- = \log(-\alpha/(1-\alpha)) - 1 \quad (13a,b)$$

If in a load-step in positive (or negative) curvatures softening occurs then the value of M_p^- (or M_p^+) for the section should be set equal to the value of the section resisting force at the end of softening load-step.

Use of conventional integration schemes for numerical integration of the rate equations engenders the problem of error propagation (e.g., Kulkarni et al. [5]). To avoid this problem direct integration scheme (assuming constant $h(z)$ and η and noting that n is assumed to be equal to one) which gives exact solution of the rate equations and prevents the error propagation, is used in this study.

5. NON-LOCAL FIBER ELEMENT MODEL

Fiber models can be classified in two main groups: stiffness based and flexibility based models. Due to interpolation of displacement by Hermitian interpolation functions in stiffness based models, curvature distribution is linear even at large curvature localization, where the actual curvature distribution is nonlinear. Following Mahasuverachai [6] in flexibility based models, the nonlinear curvature distribution can be approximated using variable flexibility interpolation functions, as follows

$$\Delta \mathbf{d}(x) = \mathbf{f}(x) \Delta \mathbf{q}; \Delta \mathbf{D}(x) = \mathbf{b}(x) \Delta \mathbf{Q}; \Delta \mathbf{Q} = \mathbf{F}^{-1} \Delta \mathbf{q} \quad (14a,b,c)$$

Now using equations (14) the section displacement vector $\mathbf{d}(x)$ will be

$$\Delta \mathbf{d}(x) = \mathbf{f}(x) \mathbf{b}(x) \mathbf{F}^{-1} \Delta \mathbf{q} = \mathbf{a}(x) \Delta \mathbf{q} \quad (15)$$

where $\mathbf{d}(x)$ denotes section displacements vector, $\mathbf{f}(x)$ is section flexibility matrix, $\mathbf{b}(x)$ is the force interpolation vector, \mathbf{F} is the element flexibility matrix, \mathbf{q} is the element nodal displacement vector, $\mathbf{a}(x)$ is equivalent variable displacement interpolation vector, \mathbf{D} is the section resisting force vector and \mathbf{Q} is the element nodal resisting force vector

In this study the fiber element model introduced by Neuenhofer and Filippou [7] is used. This model is able to satisfy equilibrium along member length even when softening occurs. By some changes in the representation, the fiber element model of Neuenhofer and Filippou is described in the following. After evaluation of the element nodal displacements, following conventional procedure in the stiffness based finite element, the member nodal forces in iteration i is obtained as

$$\Delta \mathbf{Q}^i = (\mathbf{F}^{i-1})^{-1} \Delta \mathbf{q}^i \quad (16)$$

Now the section displacements, considering the section unbalance forces at the end of iteration $i-1$, is calculated as

$$\Delta \mathbf{D}^i(x) = \mathbf{b}(x) \Delta \mathbf{q}^i + \mathbf{b}(x) \mathbf{Q}^{i-1} - \mathbf{D}^{i-1}(x) \quad (17a)$$

$$\Delta \mathbf{d}^i(x) = \mathbf{f}^{i-1}(x) \Delta \mathbf{D}^i(x); \quad \mathbf{d}^i(x) = \mathbf{d}^{i-1}(x) + \Delta \mathbf{d}^i(x) \quad (17b,c)$$

where the member nodal resisting force is determined using the variable displacement interpolation function $\mathbf{a}(x)$ as follows

$$\mathbf{Q}^{i-1} = \int_0^l \mathbf{a}^T(x) \mathbf{D}^{i-1}(x) dx \quad (18)$$

In this study the Gauss-Lobatto integration scheme is used for numerical integration. In this scheme two integration points coincide with member end sections, where significant inelastic deformation usually takes place.

If the prescribed fiber element model is used in softening problems, there will be spurious mesh sensitivity. The slope of the descending branch of the load-displacement curve will be strongly dependent on the discretization of the model used in the analysis. Increase in number of the Gauss points or the number of the elements, results in steeper descending branch. There are a number of ways to obtain objective finite element results. The non-local models provide simple and acceptable solution to overcome this problem. Two non-local models are used in this study, as is described in the following.

(1) Based on the concept of crack interactions, Bažant [1] suggested a general non-local model in which only the inelastic stress increment was evaluated non-locally and the elastic stress increment evaluation remained local. In this method after local evaluation of stress increment at each point, the calculated stress increment decomposed into elastic and inelastic components. For a flexural element this formulation becomes

$$\Delta M_i = EI \Delta \varphi + \Delta M_{in} \quad (19)$$

Where ΔM_l denotes the increment in the section resisting force evaluated locally, EI is section stiffness, $\Delta\phi$ is increment in the section curvature, ΔM_m is the inelastic increment in the section resisting force. After determination of ΔM_m the spatial average of the section inelastic moment increment $\langle \Delta M_m \rangle$ is determined by

$$\langle \Delta M_m(x) \rangle = \frac{1}{L} \int_L \Delta M_m(x, r) w_a(r) dr \quad (20)$$

where integration is done along the element length and w_a shows the weight function used for spatial averaging. Now the non-local increment in the section moment is calculated

$$\Delta M = EI\Delta\phi + \langle \Delta M_m \rangle \quad (21)$$

Non-local formulation defined above is used only when there is virgin loading otherwise the usual local formulation will be applicable. This formulation has two advantages over the non-local damage model. The first is the generality of the formulation that makes it possible to use the formulation in conventional finite element programs with little changes in algorithm and also for any type of constitutive equation. The second advantage is that due to local evaluation of ΔM the unloading and loading criteria is also local.

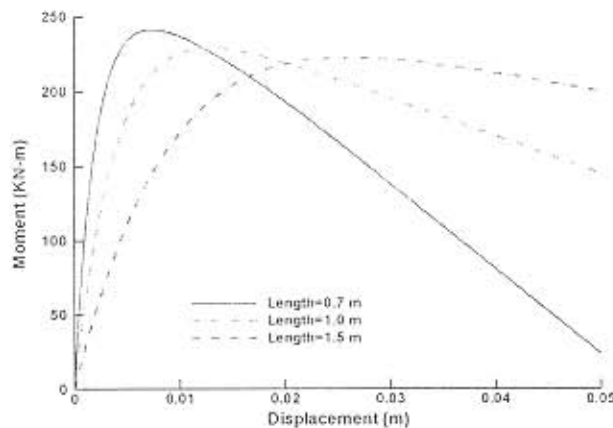


Figure 2. Effect of the moment gradient on the behavior of the non-local formulation

The weight function is

$$w_a(r) = (1 - r^2 / R^2)^2; |r| \leq R \quad (22)$$

Where R is interaction radius related to the characteristic length l_{ch} . For flexural element, R is equal to $0.9375l_{ch}$. Outside of the interaction radius, w_a becomes zero. As discussed by Pijaudier-Cabot and Bazant [8], use of Euler hypothesis in beams requires that the averaging length be greater than or equal to the beam depth. In this study the value of l_{ch} is taken equal to the beam depth. It should be mentioned that this length is also approximately equal to the length of plastic hinges of flexural members. There is no difference between local and non-local evaluation of the axial resisting force due to assumption of elastic constitutive equation for axial load.

(2) As loading process continues the interaction zone becomes thinner. If the unloading stiffness is used in definition of elastic moment increment then the decrease in the unloading stiffness due to stiffness degradation can also decrease the interaction between neighboring points. Taking this into account Jirasek and Bažant [4] suggested the use of unloading section stiffness $(EI)_u$ instead of EI of equations (19) and (21).

Application of the aforementioned formulations for flexural softening problems shows that the behavior of the models, in the presence of substantial moment gradient in the softening zone, is not satisfactory. The performance of the models improves, as the moment field tends to be homogenous. To illustrate this problem consider cantilever beams with different lengths, same characteristic length and same section behavior. It is assumed that the ultimate strength of the section is 200 KN-m. As the length of the beams increase the moment gradient along member length reduces. Figure 2 shows the behavior predicted by the first non-local model. The behavior that is predicted by the other formulation is almost the same.

6. PROPOSED NON-LOCAL MODEL

Due to the presence of moment gradient in the practical beam-column elements, to be able to obtain a good estimate of element response, there is a need for a non-local model for the cases where there is appreciable moment gradient. For these cases the following proposed non-local model gives promising results. In this model (after initiation of softening) the section moment increment is decomposed into two components as follows

$$\Delta M_l = \frac{(EI)_t}{(EI)_u} \Delta M_l + \Delta M_2 \quad (23)$$

where $(EI)_t$ is the tangential stiffness of the section and $(EI)_u$ is the unloading stiffness at peak moment. Now the non-local moment increment of the section will be

$$\Delta M = \frac{(EI)_t}{(EI)_u} \Delta M_l + \langle \Delta M_2 \rangle \quad (24)$$

where $\langle \Delta M_2 \rangle$ denotes the spatial average of ΔM_2 .

Initial stiffness method is used for iterative solution of nonlinear systems of equations. Energy increment in iterations is used as convergence criteria.

7. VERIFICATION OF THE NUMERICAL MODEL

The numerical results are compared with experimental data to assess the capability of the model. Specimen A-3 of Sheikh and Yeh [9] and specimen No. 2 of Watson and Park [12] are selected for comparison due to the observed softening in their behavior. The first specimen was tested under monotonic loading. It was subjected to constant axial load, followed by application of third point loads that results in constant moment region in the middle third part of the column. Although some sections adjacent to the load application points, located outside of the constant moment region, are also going into softening, the softening zone is a region with relatively

homogenous moment field. The second specimen was tested under cyclic loading. After application of axial load, the specimen was loaded laterally through a central stub, which simulates the beam-column connection. So the critical sections of the column are the sections adjacent to the central stub where there is substantial moment gradient. Table 1 gives details of the specimens.

Table 1. Details of verification specimens

Specimen Designation	Conc. f'_c (MPa)	Longitudinal steel		Transverse steel		Axial Load (KN)	Gross Dim. (mm)	Core Dim. (mm)
		Number and size	f_y (MPa)	Size and Spacing (mm)	f_y (Mpa)			
A-3 (Sheikh & Yeh [10])	32.4	8#6	525	#3 @108	498	1840	305x305	267x267
Unit No.2 (Watson & Park [12])	44.0	8Φ16	446	Φ8 @78	360	2110	400x400	366x366

8. SPECIMEN A-3 OF SHEIKH AND YEH

The section calculated moment-curvature is shown in Figure 3. As can be seen the differential model is able to approximate the calculated moment-curvature very well.

Figure 4 shows the result of the local formulation using three elements and different number of the Gauss points per element. The effect of imperfection is also shown in the figure. The imperfection is introduced by increasing the strength of the center element in order of 0.1 percent. In addition to mesh sensitivity, the local formulation is sensitive to the imperfection. In fact the local model is so sensitive that in some cases change of the numerical procedure can result in triggering different descending branches. This shows that the local model is not reliable for predicting the softening behavior.

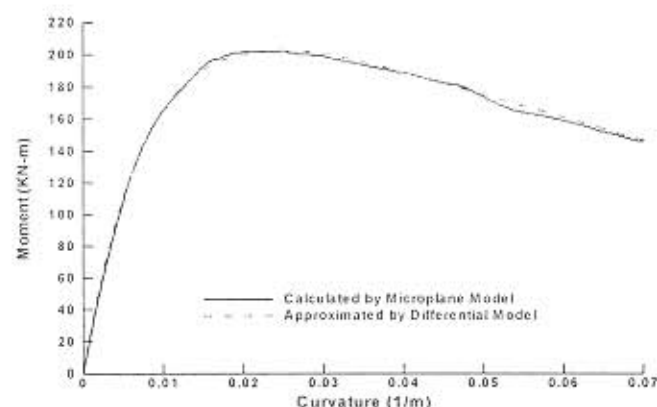


Figure 3. Moment-curvature of specimen A-3 (Sheikh and Yeh [10]) calculated by micro-plane model and approximated by hysteretic differential model

Figures 5 and 6 show the element behavior calculated by different non-local models using three elements. For modeling of each third part of the specimen an element is used. While the slope of the descending branch in the local model is strongly dependent on the number of the Gauss points, in non-local models mesh dependency is effectively suppressed. However the predictions of three non-local models are different. The non-local formulation (2) yields the best predictions. Note that the *member* resisting force calculated by non-local formulations is slightly larger than the *section* resisting force calculated using section properties (Figure 3). This increase in the resisting moment is due to existence of moment gradient in a part of the softening region and malfunction of the non-local models.

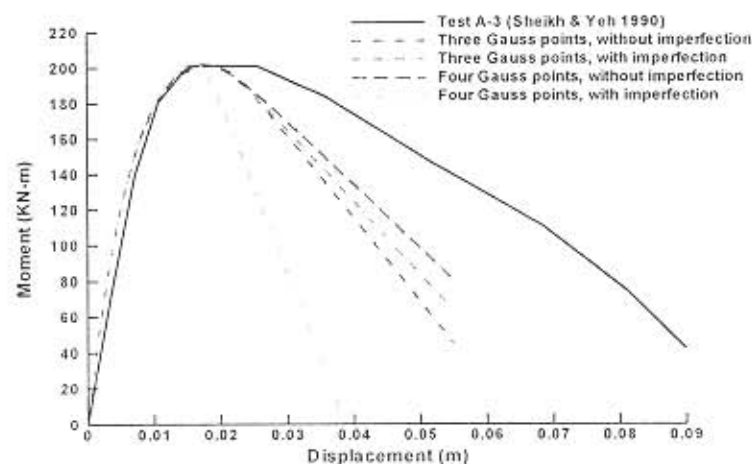


Figure 4. Mesh dependency and sensitivity to imperfection in local formulation

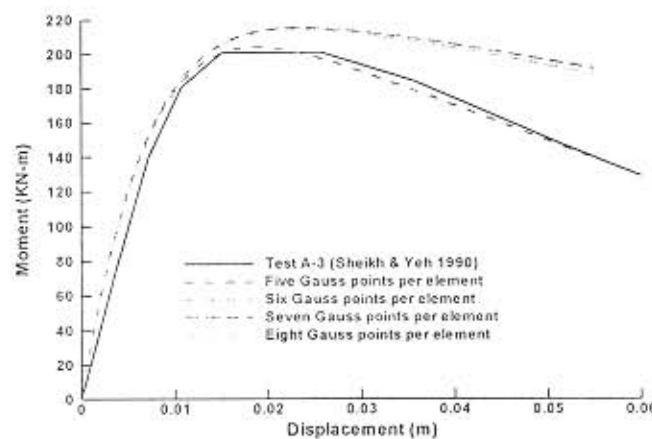


Figure 5. Moment-displacement of specimen A-3 (Sheikh and Yeh [10]) evaluated by non-local formulation (1)

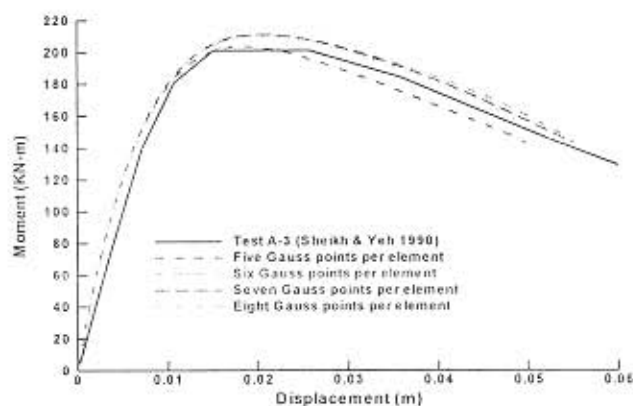


Figure 6. Moment-displacement of specimen A-3 (Sheikh and Yeh [9]) evaluated by non-local formulation (2)

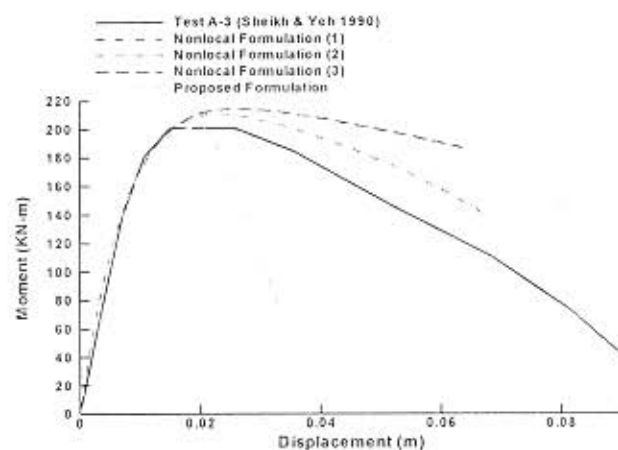


Figure 7. Moment-displacement of specimen A-3 (Sheikh and [9]) evaluated using low order elements with different non-local formulations

To compare the behavior of low order and high order elements in Figure 7 the test specimen is modeled using four elements per each third part of the specimen with three Gauss points per element. The proposed non-local formulation in this case does not give good results. In this case in which the moment field is relatively homogenous, the non-local formulations with low order elements give the results similar to those of high order elements.

9. SPECIMEN NO.2 OF WATSON AND PARK

The calculated section moment-curvature is shown in Figure 8. For this specimen due to existing moment gradient the behavior of high order elements is not as good as that observed in specimen A-3. Therefore for this specimen, elements with two Gauss points are used.

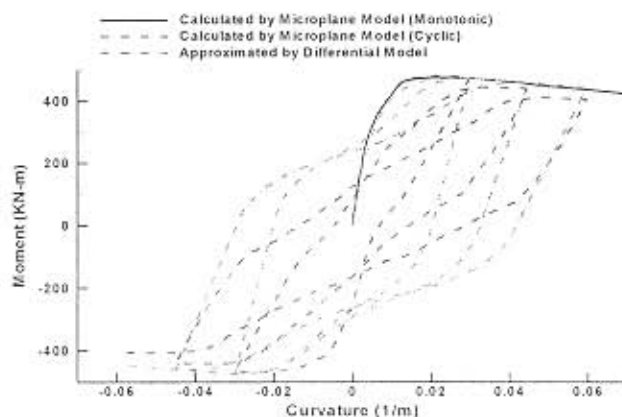


Figure 8. Moment-curvature of specimen No.2 (Watson and Park [12]) calculated by microplane model and approximated by hysteretic differential model

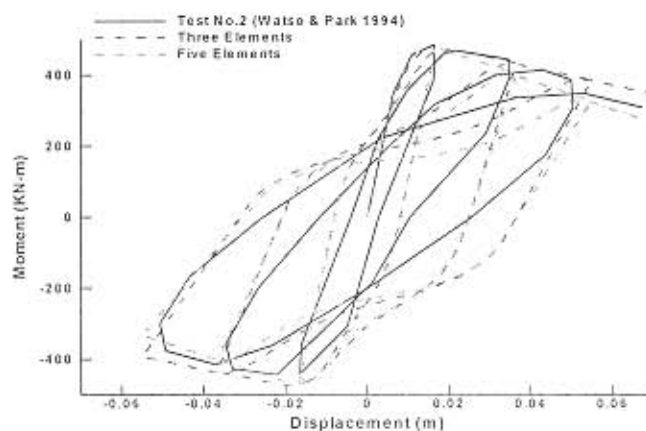


Figure 9. Moment-displacement of specimen No.2 (Watson and Park [12]) evaluated by local formulation

Figure 9 compares the result of the local formulation using three and five elements with two Gauss points per element with experimental results. As can be seen there is strong mesh dependency in the observed results.

Figure 10 shows the specimen moment-displacement evaluated by different non-local models using five elements. To be able to compare all models only the resulting monotonic moment-displacement for different models is shown. All of the non-local models suppress mesh dependency effectively, however except for proposed model there is essential increase in the predicted peak moment and resulting behavior is incorrect.

The specimen moment-displacement evaluated using proposed non-local formulation is shown in the Figure 11. Mesh dependency effectively eliminated and the resulting behavior gives a good approximation of the actual behavior.

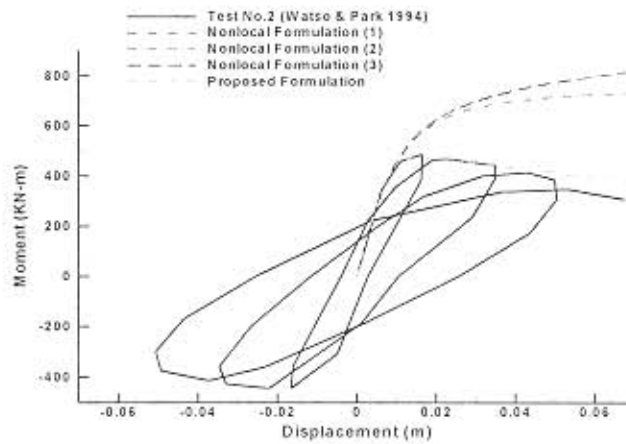


Figure 10. Moment-displacement of specimen No.2 (Watson and Park [12]) evaluated by different non-local formulations

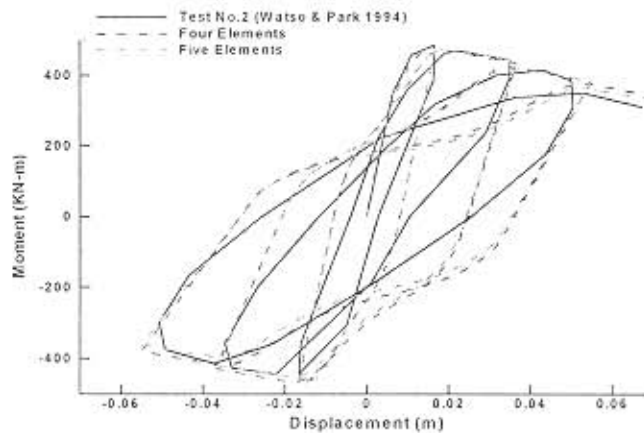


Figure 11. Moment-displacement of specimen No.2 (Watson and Park [12]) evaluated by proposed non-local formulation

CONCLUSIONS

The section moment-curvature is evaluated using micro-plane and bilinear elasto-plastic models for concrete and reinforcements, respectively. The micro-plane model gives good simulation of the concrete behavior especially for passive confinement by transverse reinforcements. To improve numerical efficiency in frame analysis, the calculated behavior is approximated using a proposed differential hysteretic model. Deficiency of the conventional differential models in cyclic loading that enters softening is investigated and a new model proposed. To avoid mesh dependency non-local measures available in the literature investigated. It is shown that in the presence of appreciable moment gradient in softening region, the non-local models gives unacceptable results. A non-local model developed that gives promising

results in the presence of the moment gradient. This study shows the need for a non-local model that is equally applicable for homogenous and non-homogenous moment fields. The performance of low and high order elements in the presence of homogenous and non-homogenous moment fields is investigated. The proposed method can be used to investigate the effect of possible softening in the overall response of the RC frame structures subjected to earthquake excitation and assess the applicability of the Codes seismic provisions to the cases where there is softening possibility.

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Notations

E	= Young modulus of elasticity of concrete
EI	= section stiffness
$(EI)_u$	= unloading section stiffness
$(EI)_{up}$	= unloading section stiffness at peak moment
f_{cr}	= tensile cracking stress
$h(z)$	= pinching function
M	= section resisting moment
n	= parameters of differential hysteretic model
w_a	= weight function used for spatial averaging
z	= section hysteretic curvature
$\mathbf{a}(x)$	= displacement interpolation function vector
$\mathbf{b}(x)$	= force interpolation function vector
$\mathbf{d}(x)$	= section displacement vector
$\mathbf{D}(x)$	= section force vector
$\mathbf{f}(x)$	= section flexibility matrix
\mathbf{F}	= element flexibility matrix
\mathbf{q}	= element displacement vector
\mathbf{Q}	= element force vector
α	= ratio of post-yield to pre-yield stiffness
β, γ	= parameters of differential hysteretic model
$\delta_1, \delta_2, \delta_{10}$	= constants of pinching function
ε_{cr}	= tensile cracking strain
η	= stiffness degradation parameter of differential hysteretic model
λ	= constant
ρ	= degree of efficiency of passive confinement
φ	= dimensionless section curvature